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Simulation models for Economics

Final thesis of the course:

Collusion and Price Competition

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Introduction

Bertrand Paradox and price competition are two well-known concepts in microeconomics and industrial economics, presented in almost all the textbooks of these disciplines, and always presented with respect to the Cournot analysis, which refers to the quantity competition; this is also the starting point of this model, which, thanks to the useful tools that agent-based models can offer, aims to give a representation of what happens in markets in which the competition is based on price. The model aims to represent the effects that collusion and price competition bring to the satisfaction of the consumers.
Inspired by the theory of the so-called "Bertrand Paradox" (1883) and then extended to the concept of collusion, as a consequence of the results which the paradox gives us, it reflects on how it can be possible in the real world to have a price equal to the marginal cost in a market which has only two firms; thinking from a firm point view seems not plausible that the two firms would give up so easily a consistent amount of profits.
Therefore, we analyze the concept of tacit collusion, which can be considered as a key point of the industrial organization theory; famous economists as Rotemberg, Saloner, Haltiwanger, Harrington, Green and Porter have gone deeply in this topic proposing different models, considering differently the way the collusive behavior arise. Among the different scenarios, we mostly consider the case of the infinite horizon. The game is repeated but this would not have allowed us to infer different conclusions from the one-shot game without stressing the importance of the interest rate and the discount factor.
Nice to notice are the two different perspectives which this analysis can bring, from the firms point of view and from the consumers one.
Therefore, we introduce the satisfaction of the buyers, identified with their price preference as a simplification of their demand curve, to understand if it is possible that social welfare wins in this system or if "unhappy consumers" is the only consequence of the collusive or competitive behavior of the firms.

Bertrand Paradox (1883)

In Economics and Commerce “The Bertrand Paradox” is a model used to explain the competition in price between firms; usually the results from this model and the interaction of the firms are compared with the ones coming from the Cournot model, which differently presents a quantity competition.

The model starts from a simplification, presenting only two firms which produce homogenous commodities or, we can say, “non-differentiated”, so they can be considered perfect substitutes in the consumers’ utility functions. Therefore is just a matter of which firm charges the lowest price [min p1,p2]. The firms do not set the quantity.

Another key assumption is that there are no capacity constraints, hence the firm always supplies the demand it faces.

In the case in which the same price is charged by both firms, it is crucial to understand how the distribution of the consumers works; at this purpose the model assumes that each firm supplies half of the market demand.

In addition, the two firms choose their prices simultaneously and non cooperatively; so considering the last assumption, which consists in equal and constant marginal costs, the conclusion of the model is that each firm will choose the competitive price (equal to the marginal costs).

This result comes from the possibility of either firm to lower its price, even just a little (epsilon), gaining substantially the whole market and raising deeply its profits.

The reason of why it is called a “paradox” is properly the Nash equilibrium of economic profit equal to zero which the model reaches: so, paradoxically, even a duopoly would suffice to restore the perfect competition situation, and, clearly, it does not just seem very plausible. What intuitively seems plausible is that few firms in industries can succeed in raising the price above its competitive price.

Also some empirical works have shown the evidence of positive profits in industries with just two competitors.

How to solve the Bertand Paradox?

The Bertand Paradox can be solved in three ways, so by relaxing one of the three main assumptions of the problem:

- No capacity constraints
- Homogeneous goods
- One-shot game

The first way to solve the Bertand Paradox is the so-called “Edgeworth solution”, which consists in the introduction of capacity constraints. If we put a constraint in the production size, for a firm it isn’t useful to go into an extreme price competition, since they wouldn’t have the possibility to satisfy all the market demand. The second way is the product differentiation, which rules out the important assumption of the perfect substitutability of the firms’ products. Consumers don’t consider anymore the price as the unique parameter during choice of buying one good or another, so differentiated products in a way relax the price and they allow to escape theBertandParadox.

In the NetLogo model we concentrate our attention on the third way, the ***dynamic price competition***. If we rule out the assumption of a “one-shot game” and we introduce the possibility to a firm to react, it becomes possible to find results different form the Bertand equilibrium. In fact, the firm has to take into account not only short-run profits, but also eventual long-run losses given by a fail in the collusion with the competitor. Moreover, the model is based on the *infinite horizon* assumption ($T= +\infty$). The finite horizon case is not so interesting in our purposes, since it doesn’t show anything different from the “one-shot game”.

The model: *tacit collusion in the infinite horizon case (*$T= +\infty$*)*

The model is based on the T+1 repetitions of the Bertand game, and the aim is to verify how the price war takes place in a non-cooperative duopoly case.

Assumptions:

- Two firms (1,2) produce homogeneous goods
- Constant marginal costs (*c*)
- Each firm supplies the demand it faces in every period

- In each period consumers buy from the firm with the lowest price; if the price is the same, the firms supply an equal part of the total market (1/2 each one)
- The firms set the price noncooperatively and simultaneously, but they take into account the history (H_t) of the previous prices

$$H_t = (p_{10}, p_{20}; \ldots\ldots; p_{1,t-1}, p_{2,t-1})$$

An important role in the model is covered by the parameter δ, the discount factor

$$\delta = \frac{1}{1+i}$$

If it is close to 1, it represents the low impatience of the firms. In the model, each firms maximizes the present discounted value of each profits, so

$$\sum_{t=0}^{T} \delta^t \Pi^i (p_{it}, p_{jt})$$

Then, we have to define the price strategy pursued by the firms, and in this case we consider the so-called *trigger strategy*: " a single deviation triggers a halt in the cooperation"[1]. This symmetric price strategy is built in this way: if the firm 1 sets the monopoly price p^m in the first period t=0, it will set again the monopoly price in t=1 only if firm 2 has set the monopoly price too in t=0; if this not occurs, so if firm 2 has cheated by reducing the price in order to have the full profits of the market, firm 1 will set the competitive price forever *(P=c)*.

The symmetric trigger strategy is a point of equilibrium if the long-run losses from cheating are greater than short-run gains of getting all the profits of the market

$$\frac{\pi^m}{2}(\delta + \delta^2 + \ldots) \geq \frac{\pi^m}{2}$$

Which implies $\delta \geq 1 - \delta$ and $\delta^B \geq \frac{1}{2}$

So, if δ is high there is an incentive for the firms to set the price in a non-cooperative way, while the opposite occurs if the value of the discount rate is below the threshold one.

[1] Jean Tirole, *The Theory of Industrial Organization*, MIT Press 1988, p. 246

The formal model

In order to explain the main relations of the model, we have to define three types of profits that we find in the formula:

- Profits of collusion, π^{C}
- Profis of deviation (cheating), π^{D}
- Profits of non-collusion, π^{NC}

We define the profits of the deviation minus the profits of collusion as

$$(\pi^{C} - \pi^{NC})(\delta + \delta^{2} + \ldots) \geq \pi^{D} - \pi^{C}$$

Which implies

$$(\pi^{C} - \pi^{NC})\delta \geq (1-\delta)(\pi^{D} - \pi^{C}) \quad \text{and} \quad \delta \geq \frac{\pi^{D} - \pi^{C}}{\pi^{D} - \pi^{NC}}$$

In the Bertrand game we set the profits of non collusion equals to zero ($\pi^{NC} = 0$), since it is irrelevant for the model in order to explain collusion. So, in the case of cheating the firm gets all the monopoly profits ($\pi^{D} = \pi^{m}$), and the profits of collusion are half of the full market ($\pi^{C} = \frac{1}{2}\pi^{m}$).

Finally we have that $\delta \geq \dfrac{\pi^{D} - \pi^{C}}{\pi^{D} - \pi^{NC}}$ which implies $\delta^{B} \geq \dfrac{1}{2}$

THE MODEL

To Setup

The setup button is the starting point of the model, it defines the key elements which will interact subsequently.
These elements are clearly firms and consumers: they are represented by their own setup functions, SETUP-FIRMS and SETUP-CONSUMERS.

```
globals [ price-war marginal-cost competition-price decreasing-price
    delta delta-threshold competition interest-rate
    collusion-price  number-moving-consumers firm-profits
    number-happy-consumers number-unhappy-consumers]
breed [ firms firm ]
breed [ consumers consumer ]

consumers-own [ oldcustomer happy? price-preference change-firm ]

to setup
  clear-all
  setup-firms
  setup-consumers
  reset-ticks
end
```

To complete the analysis of the setup button we have to underline other two functions presented in the setup button: the *clear-all* function and the *reset-ticks*, which are typical of NetLogo models; the first allows to create every single time a different world, combining the effects of clear-globals, clear-ticks, clear-turtles, clear-patches, clear-drawing, clear-all-plots, and clear-output. The second reset the ticks at the zero level; they are considered as a unit of time and increase over time, in the model can be considered as years or months. Consumers and firms will adapt their behaviors every tick, so in the experiment's part will be interesting to notice when the various equilibria will be reached.

Now we can step into the two most important functions that the setup button calls.
Setup-Firms: it allows to create the firms, more precisely four, and the reason for this kind of choice is due to the necessity to follow the assumptions

```
to setup-firms
  create-firms 4
  [ set shape "factory" set color 7 set size 5 ]
   ask firm 0 [setxy 8 8]
   ask firm 1 [setxy -8 8]
   ask firm 2 [setxy 5 -8]
   ask firm 3 [setxy -8 -8]
   ask firms [ ask patches in-radius 7
      [ set pcolor 75 ] ]
```

of the Bertrand model, which assumes a duopoly; so the two firms at the bottom will interact together and the two at the top side of the world will interact only with themselves.
The double one-to-one relationship is not just a mere repetition, the differences are the markets which are represented by the patches of radius 7 and color turquoise, which can be completely separated or can create an intersection. Where the markets intersect, the consumers have the possibility to choose from which firm buy, which needless to say is the firm that offers the lower price or in the case in which no firm offers a lower price they can anyway choose randomly.
In a different way the behavior of consumers is affected by the patches at the top of the screen, which are separated and not allow to anyone of the consumers on them to pass from one to another. Buyers, therefore, are obliged and have to take passively the price as given, this is clearly a strong limitation but can also lead to interesting considerations basically on the consumers satisfaction.

Graphically the firms are placed as we said before, not randomly but in the four quadrants of the the cartesian space, on which the Netlogo world is based. The shape chosen is the factory, the color is set as grey and the factories are set bigger than the consumers just for graphical and logic reasons, in the attempt to recreate the reality.
In the following lines of the code are defined also the characteristics of the links which will be created next in the collusion case: shape, color and thickness are set again for graphical reasons.
The setup is also composed by an *output window*, which initially reflects only the level of three main variables: collusion price, competition price, defined as "initial price", and the marginal cost.
We have now to consider the two most important functions called by the setup-firms function***: set-price*** and ***set-marginal-cost***, which, as we said before, define essentially the key variables which regulate all the relationship that will happen in the following steps.

```
to set-price
  ask firms[
    ifelse Collusion
    [set collusion-price InitialPrice
      ask firm 2 [ create-link-with firm 3 ]
      ask firm 0 [ create-link-with firm 1 ]
    ]
     [set competition-price InitialPrice
     ] ]
end
```

Starting from the price definition, fundamental is the *ifelse* condition which allows to activate the switch in the interface, which selects what case is taking place (collusion or perfect competition). Actually there are no significative differences between the two cases, and the price is set through a slider, which assigns a value between 0 and 30. The only difference is the creation of the links which underline the presence of a collusive behavior.

Furthermore, we have to consider the ***set-marginal-cost function***, which sets it through the use of another slider, which assumes values between 0 and 20; conditions on the possibility that the marginal cost can assume values lower than the price are necessary to avoid mistakes and unexplainable situations; therefore, in both cases if the price is lower than the marginal cost at each firm is assigned a label with marks the negative situation in which firms are clearly in default, not being able anymore to face the costs.

Another label is set in the case in which the firms throughout perfect competition reach the competitive price, which is equal to the marginal cost.

Setup-consumers: this function defines the other side of the story, the consumers one; the number of total consumers present in the simulation is the first thing which is set; the lower limit is 4, allowing that just for the beginning each firm has at least one consumer. This first four consumers are exactly placed in definite coordinates; the idea is to identify consumers which was *old customers* of the firm and so are strictly related to it by assumption. Differently, the number can be increased until reaching the limit of 100 consumers that will be spread randomly between the firms, more exactly on the patches which each firm creates around itself, excluding the possibility of being placed out of the markets.

```
to setup-consumers
  create-consumers HowManyConsumers?
  [set shape "person" set color 9.9
   set price-preference PricePreference
   ]
   ask consumer 4 [setxy 9 9 set oldcustomer 1]
   ask consumer 5 [setxy -7 9 set oldcustomer 1]
   ask consumer 6 [setxy 6 -7 set oldcustomer 1]
   ask consumer 7 [setxy -7 -7 set oldcustomer 1]
   ask consumers with [oldcustomer = 0]
   [move-to one-of firms with-local-randomness [fd random 7]]
   ask consumers with [ycor < 0 ] [set change-firm "yes"]
   ask consumers with [ycor > 0 ] [set change-firm "no"]

end
```

Clearly, also in this case for graphical reasons we have set the shape as human beings and we, indeed, set the color white.

Through the use of a slider we identify, moreover, a precious variable which is the *price preference* of each consumer; therefore the consumers want to buy what firms offer but they prefer to do it at a price which is comfortable for them. They will anyway buy the good, but if the price will be higher than their preference they will be unhappy and unsatisfied.

The last thing to notice is the *change-firm* variable, typical only of the consumers breed, it identifies what set of consumers is allowed to move from one firm to another, searching for the best price, underlying again the difference between the top and bottom of the world.

This ends the setup function and create a simplified world that allows us to make more understandable and intuitive the consequences that the Bertrand paradox and the price competition bring. The relationship between the key variables will be presented in the next part in which the *to go* button will be explained.
We'd like to stress again the importance of the switch (on/off) named "Collusion" as key element that can completely change the scenario: changing from a simple competition in price, where a price war has necessarily to arise, and the collusion case where the solution of the Bertrand paradox is introduced and, hopefully, clearly enough presented.

To Go

The "to go" procedure regulates basically all the functions of the model: breaking-the-collusion, move-consumers, perfect-competition, create-price-war-plot, create-firm-profits-plot, create-consumers-mood-plot, create-consumers-surplus-plot, create-delta-plot, check-consumer-mood.
It covers both the cases of price determination and competition, the behavior of consumers and the creation of the plots.

The button in the interface is set as a forever button, in this way the model will run continuously and as we said also before the tick function is a measure of time. We do not have imposed any limitation for the ticks, the only limitation, which is presented at the first lines of the code, is imposed on the relationship between price and marginal cost; it has to be greater to avoid, conceptually, negative profits for the firms and a continuous decrease in the price setting which would not have much sense, thinking at the reality. This is realized thanks to another *ifelse* function which stops the simulation in both cases.

Selecting "on" in the switch button, "Collusion" we will launch the breaking-the-collusion function present in the go button.

To breaking-the-collusion

```
to breaking-the-collusion
   if Collusion [cheat]
   set interest-rate random-float 1
   set delta-threshold 0.53
   set delta (1 / (1 + interest-rate) )

end
```

The function is created to define how the collusion is broke by one of the firms; in fact we have to consider that collusive behavior is not a permanent and constant situation, but it is affected by the interest rate and the discount factor. Considering their actual and future profits, firms have to be sure that long-run gains will be higher than short-run gains; or, put it differently, the long-run loss has to be greater than the short-run gain to maintain the equilibrium. Therefore we set three key elements:

- the *interest rate* as random number between 0 and 1, which is actualized and correct every tick.
- the *delta threshold* which is bounded between 0 and 1 as well, and has to be near 1 to make the collusion plausible and the cheating behavior not convenient for any of the firms.
- the *delta,* which represents the discount factor and has an inverse relationship with the interest rate. It must be compared with the threshold as in the interface presented in the plot "delta".

To cheat

Afterwards, it is the cheat function which, actually, sets the conditions for the cheating behavior; if the interest rate reaches an high level, it affects the discount factor which decreases towards the limit of 0,5, under which the collusion becomes unrealistic because short-run gains obtained by cheating will be higher than the long-run ones.

Therefore, one of the firms has the incentive to lower a bit the price and getting all the profits for one period, even though it knows that in the following period the rival will set its price at the competitive price as a punishment.

Graphically the phenomenon is represented by the disappearance of the links between the firms and the changing of the color of the firm which cheat (it becomes red); the cheating firm also creates links with all the consumers in the lower markets, which, having the possibility of searching the best price, clearly, switch to the more convenient and cheating firm. This is made possible by another function, **searching-the-best**. It selects which set of consumers are able to do this and creates the links, setting thickness and color too.

Also a label is applied to the cheater and the output window is updated and it shows:

- the cheating price, which consists in the initial price minus a random number between 0 and 1.
- the script which advises the observer that a firm has cheated.

When the go button is pressed and the process is launched, without slowing the speed, everything happens rapidly, but we can observe how many ticks have been passed before a firm decide to cheat the other, breaking the collusive behavior.

```
to cheat
  if interest-rate > 0.867
  [set collusion-price (collusion-price - random-float 1)
     output-print "cheating price" output-print collusion-price
    set collusion-price MarginalCost
  ask links [die]
  ask firm 3 [set color red set label "cheater"]
  ask consumers [searching-the-best]
  output-print "A firm has cheated!" ]

end

to searching-the-best
 ask consumers with [change-firm = "yes" and ( xcor > 0)]
 [create-link-to firm 3]
 ask links [set thickness 0.2 set color red]
end
```

To perfect-competition

```
to perfect-competition
  if (not Collusion)
    [price-competition]

end

to price-competition
   ask firm 3
   [ set competition-price (competition-price - random-float 1)]
    output-print "leading price I" output-print competition-price
    ask consumers with [change-firm = "yes" and ( xcor > 0)]
    [move-to firm 3 with-local-randomness [fd random-float 6.5
    lt 360]]
   respond-price-competition

end

to respond-price-competition
  ask firm 2
  [ if competition-price > marginal-cost
    [set competition-price (competition-price - random-float 1)]]
  output-print "leading price II" output-print competition-price
   ask consumers with [change-firm = "yes" and ( xcor < 0)]
    [move-to firm 2 with-local-randomness [fd random-float 6.5
    lt 360]]
end
```

The command "to perfect-competition" and the code linked to this function represent an important part of the model, since they define the *non-collusion case* and the price-war among the firms of our market.
The mechanism through which we have built the perfect competition is made of two different functions: "to price-competition" and "to respond-price-competition".

- The role of the command "to perfect-competition" is just to specify that we are in the non-collusion case and to launch the price-war, that starts with the function ***to price-competition***. The main idea is that the firm 3, in the lower left side of our model, is able to set the new price in the market, which is lower than the initial price chosen with the slider; this new price is called "competition-price" and it is printed in the monitor on the right of the world with the name "leading price I". In order to make the model more realistic, the firm sets the competitive price by subtracting the initial level by a random number between 0 and 1.
  Before the reaction of the rival in the market, we can see a change in the consumers' behavior due to the lower price; in particular, we refer to the consumers who are in the area of influence of the firm 3 and of the firm 2, since the two circles are intersected. So, consumers characterized by the function "change-firm = yes" move to firm 3, since they can buy the homogeneous product of the market at a lower price; clearly, who is already in the area of influence of the leading company doesn't change his position.

The number of moving consumers is printed in the monitor on the right of the world.

- In a context of perfect competition, the price-war takes place until the price is equal to the marginal cost of the product, and also in this case a firm responds to the new competitive price by lowering it. This repeated game is represented in the model thanks to the function ***to respond-price-competition***, which is built on the same mechanism seen for the command "to price-competition": the firm 2 sets a new price by subtracting from the firm 3's "competition-price" a random number between 0 and 1. As in the previous case, the consumers in the area of influence of the firm 3 move to the area of influence of the firm 2, where they can buy the product at a lower price

The process takes place repeatedly until we reach the perfect competition condition, so *price=marginal cost*. From that moment forward the price will be the same and there will be no need for the consumers to move from one firm to another.

To check-consumers-mood

```
to check-consumer-mood
  ifelse collusion
  [check-consumer-mood-collusion]
  [check-consumer-mood-perfect-competition]
end

to check-consumer-mood-collusion
  ask consumers
  [ ifelse (price-preference >= collusion-price)
    [set happy? true set color blue]
    [set happy? false set color red]]
     set number-happy-consumers count consumers with [happy? = true]
     set number-unhappy-consumers count consumers with [happy? = false]
end

to check-consumer-mood-perfect-competition
  ask consumers
  [ ifelse (price-preference >= competition-price)
    [set happy? true set color blue]
    [set happy? false set color red]]
     set number-happy-consumers count consumers with [happy? = true]
     set number-unhappy-consumers count consumers with [happy? = false]
end
```

The model that we have built up tries to capture not only the firm's behavior, but also the effect on the consumers' welfare; this has been made possible by imposing a "price preference" of the consumers, set through a slider, and by finding if the price preference is less, equal or greater than the price of the model. In other words, we can find out if the consumers are happy or unhappy.

Also in this case, the mechanism is made of two different functions: “to check-consumers-mood-collusion” and to check-consumers-mood-perfect-competition”.

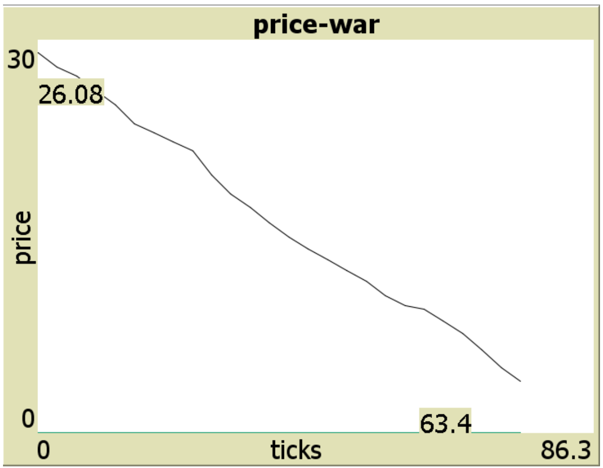
- The command “to check-consumer-mood” launches ***to check-consumer-mood-collusion*** in the case of collusion. The result of this function is that if the price preference of the consumers is greater or equal than the collusion price (“price-preference >= collusion-price) the consumers are “happy”, and so they become blue. The opposite occurs if the price preference is less than the collusion price: in this the consumers are red.
  The function is built by using the “true” and “false” command: if the statement (price >= collusion-price) is true, consumers are happy and so blue. If the statement is false, consumers are not happy and so they are red.

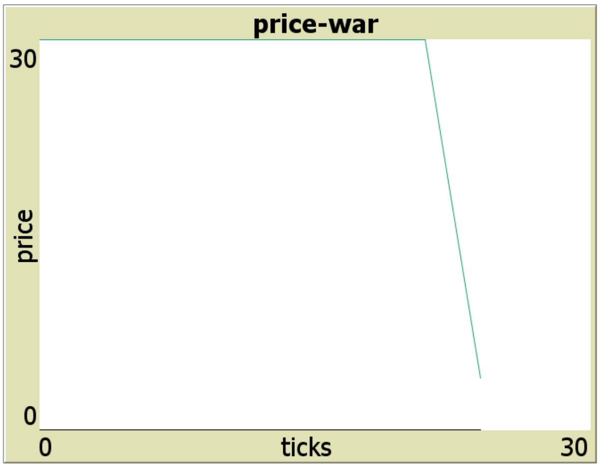
- The command ***to check-consumer-mood-perfect-competition***, launched by “to check-consumer-mood”, is built on the same mechanism of the collusion case, but with the perfect competition game. The only part of the code which differs from the previous case is the price one, since we are not interested anymore in the collusion price, but in the competition price. Therefore, if the price preference of the consumers is greater or equal than the competition price (“price-preference >= competition-price) the consumers are “happy”, and so they become blue. The opposite occurs if the price preference is less than the competition price: in this case the consumers are red.
  The function is built by using the same “true” and “false” mechanism of the previous case.

Plots

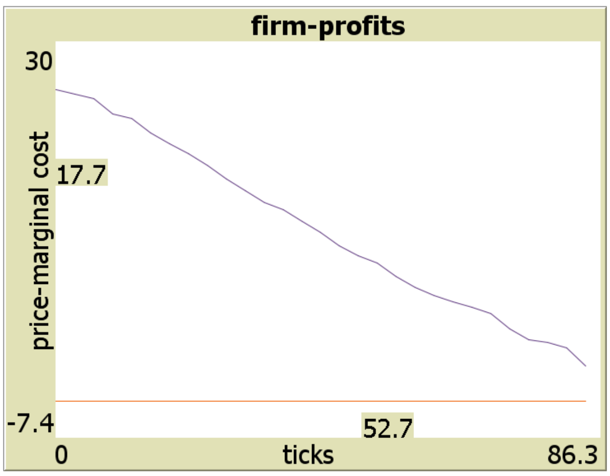
In order to make it clear how the main variables of the model work, we have created five plots that show the consequences on firms and consumers of the collusion and perfect competition.

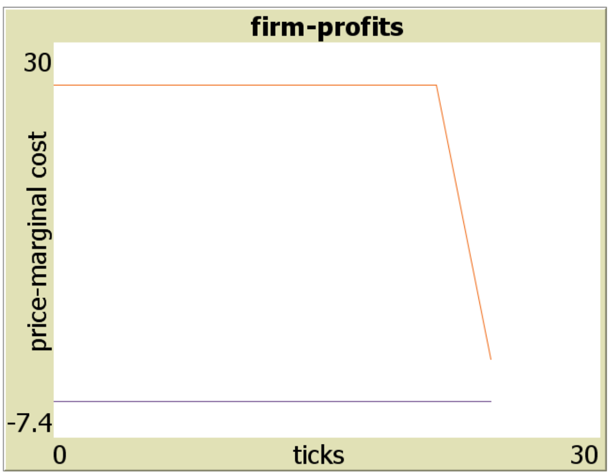
- ***To create-price-war-plot*** sets a graph in which is represented the price trend of the model, both in the case of collusion and in the case of perfect competition; clearly, if we are in the collusion case the black pen, which is referred to the "competition-price", will show a zero result; the opposite occurs with the green pen in the case of perfect competition.



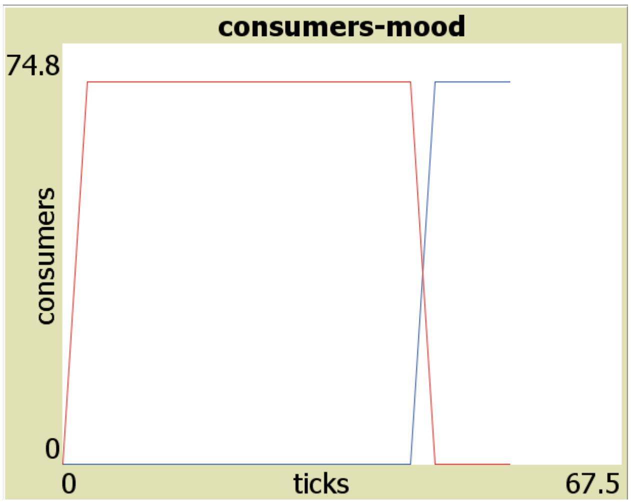


- ***To create-firm-profits-plot*** sets a graph which represents the profits of the firms on the single product sold to the consumers. It is based on the formula "profit = price – marginal-cost" and it works both in the case of collusion (orange pen) and in the case of perfect competition (purple pen).

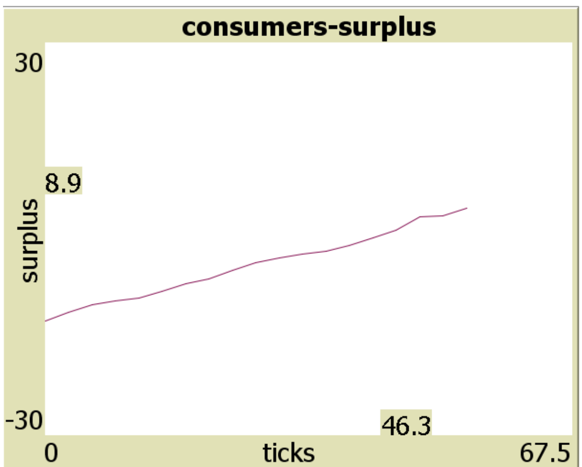


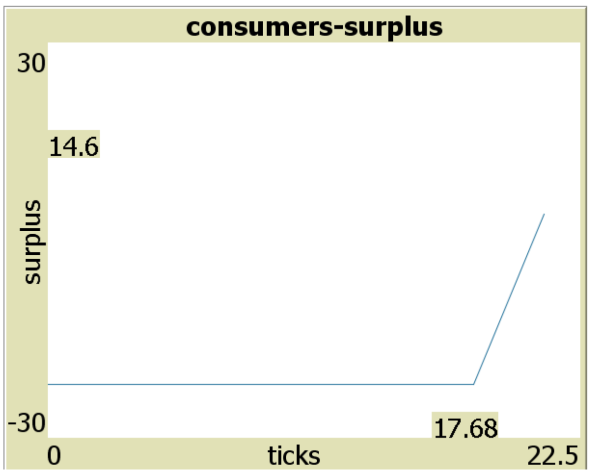


- ***To create-consumers-mood plot*** creates a graph which shows the trend of the happiness/unhappiness of the consumers, strictly related with the price that works in the market and with the price preference set through the slider. The blue-pen indicates the number of happy consumers in the model, while the red-pen indicates the number of unhappy consumers.

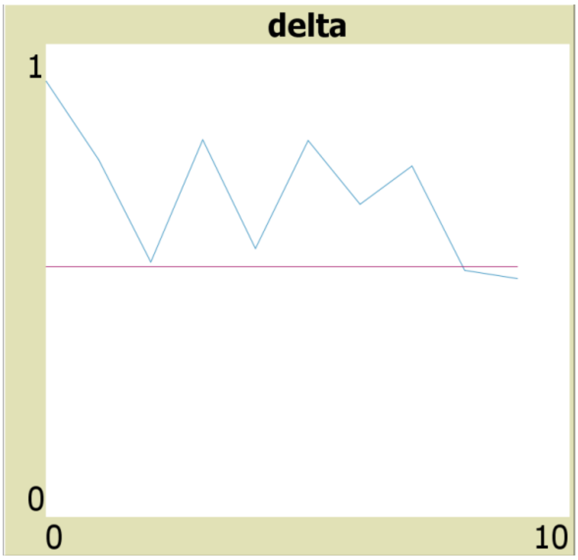


- ***To create-consumers-surplus-plot*** sets a graph which shows the consumers' welfare trend, so the benefits they are enjoying or the losses they are suffering by taking care of the price preference. The plot is built both for perfect competition (purple-pen) and for collusion (light blue-pen), and the only difference between the two cases is represented by the price level:
  "price-preference – competition-price" in the case of perfect competition and "price-preference – collusion-price" in the case of collusion.





- ***To create-delta-plot*** sets a graph which is related only to the collusion case, and it shows the delta trend in the market and the delta threshold level. As said in the previous pages, one of the firms cheats randomly from the monopolistic price when the delta level exceeds the delta threshold (set at 0.53). Coming back to the formula “delta (1 / (1 + interest-rate))”, when the interest rate is over 0,867 the delta level (light blue-pen) drops beyond the delta threshold level (fuchsia-pen), and collusion falls.



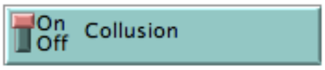

```
to breaking-the-collusion
    if Collusion [cheat]
    set interest-rate random-float 1
    set delta-threshold 0.53
    set delta (1 / (1 + interest-rate) )

end

to cheat
  if interest-rate > 0.867
  [set collusion-price (collusion-price - random-float 1)
     output-print "cheating price" output-print collusion-price
   set collusion-price MarginalCost
```

Experiments

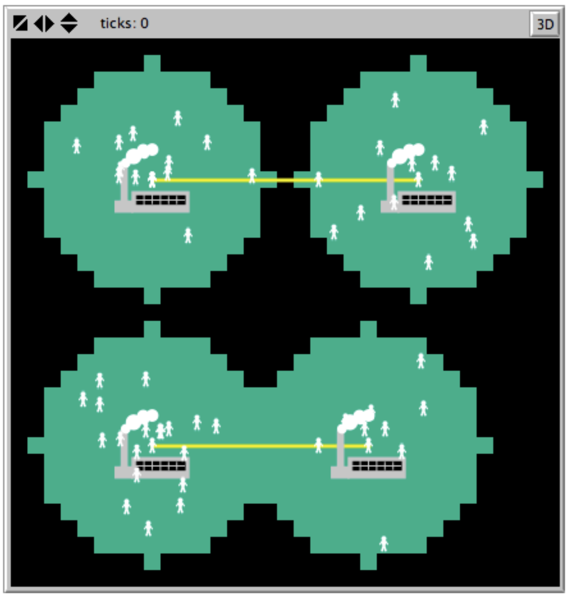
Now we can proceed to perform controlled experiments under a variety of conditions, which can be considered the objective of the whole work. The idea, therefore, is to try the two main scenarios: the collusion case and the perfect competition case.

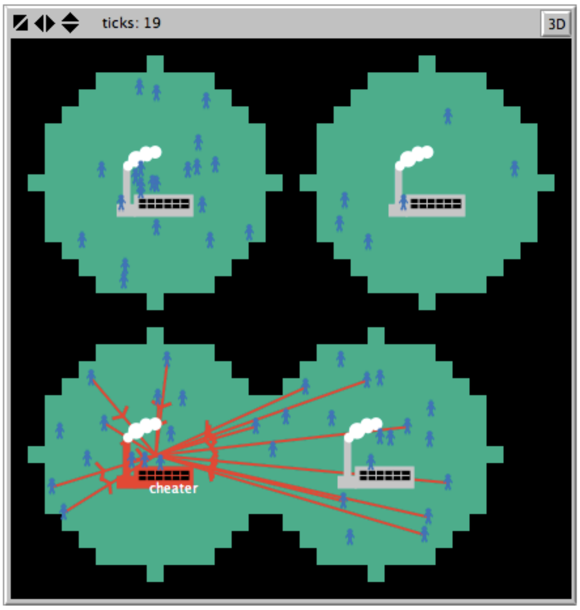


We will start analyzing the collusive behavior, so the switch button "Collusion" has to be switched on.

The sliders present in the interface are able to set the key variables which regulate the simulation. Therefore we can set the initial price, the marginal cost and price preference of the consumers; the slider which imposes the number of total consumers present in the world affects only the number of consumers which are allowed to move in the intersection of the two markets at the bottom.

What we want to observe in this experiments is the satisfaction of consumers when the firms collude and when the collusion is broke.



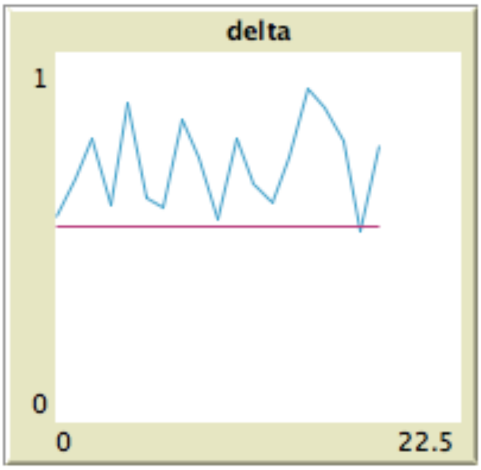


Collusion

Delta variations

The collusive behavior is based mainly on the level which the discount factor assumes, nevertheless it cannot be set through a slider, but it is set randomly in the code section; we can only observe how it changes in the plot in the low-left part of the screen.

Looking at the plot we can see how much time passed before one of the firms realize that the discount factor has became too close to 0.5 and so it can choose to lower the price a little, steal all the market and then follow the rival which will set the price equal to the marginal cost; this is commonly called the "punishment phase".

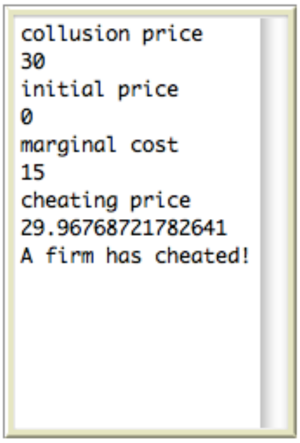


What cannot be observed is the behavior of the interest rate, but knowing that it is in an inverse relationship with the delta factor, we can easily understand that it will be increased.

Running more simulations we could observe that the time has no role in the determination of the delta; the collusion could be broken in just few ticks or, differently, the process can take a long period of time.

Interesting to notice the difference between the two parts of the world: at the top the collusion is broke, in fact the link between the firms is disappeared but the consumers cannot change and move to the more convenient firm. Anyway considering the price preference we can observe that, if it is equal or greater than the price lowered by the cheater, the number of happy consumers increase. At the bottom, otherwise, the cheater is evident and the links underline the steal of all the consumers and all the profits for one period.

Initial price and marginal cost



In the collusion case there is no repeated price war, but the price is set at the initial level, which can represent the monopoly price that is by definition greater than the marginal cost; it can be interpreted as a result of an agreement between the two firms, not as a contract, but it is more plausible a verbal agreement which can be broke in the way we have imposed before. As said in the previous pages, the price slider is bounded between 0 and 30 meanwhile the marginal cost between 0 and 20.
In a limit case in which we have “initial price = 30” and “marginal cost = 0”, the discount factor changes through time, before reaching the final result:
“collusion-price = 0”, and so “price = marginal cost”. The important mid-term result is the cheating price which appears in the output which is a bit lower than the initial price.
We can concentrate ourselves also in two other cases:
the initial situations of “P = MC” and “P < MC”.

- If we set “initial price = marginal cost”, all the collusive and cheating behavior are useless and do not happen; the considerations that we can do is that the firms are already setting the price equal to the marginal cost and profits are equal to zero, so, there are neither cheater or cheated firms. Links are already present because firms are in a sense colluding considering that no cheating can be possible. A label appears and signals that the price is equal to the cost. Then, the process is stopped because nothing else can happen.
- If we set “initial price < marginal cost”, the collusion is offset; setting the price lower than the marginal cost is not plausible because will create obvious losses. Clearly, no one of the firms would start any kind of production and this paradoxical situation is stressed by the appearance of the label “default”.

Price preference cases

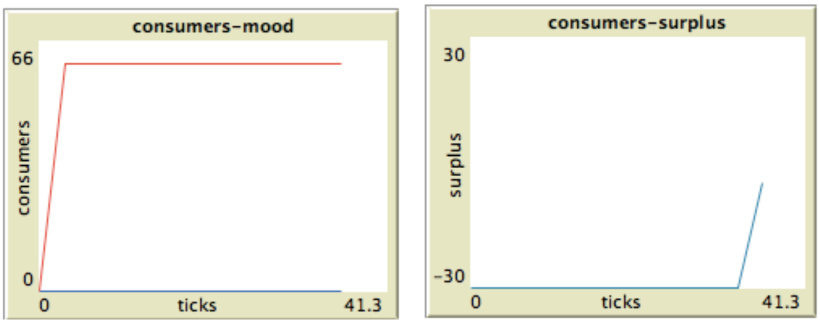
This represents the focal point of our analysis, the satisfaction and the reaction of the consumers; they are basically interested in paying the lowest price they can, so everybody, if asked to, would say that the ideal price is something near to zero. The presence of the slider which can set independently their preference allow to understand and observe how and how much time has to pass before the mood could change.

The experiment is based on the following settings:

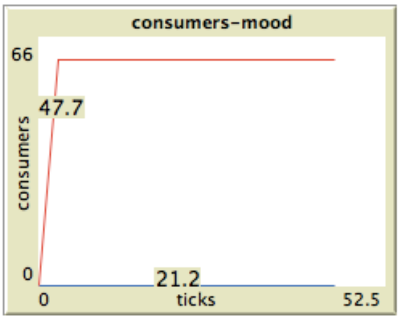
- InitialPrice = 30
- MarginalCost = 5
- HowManyConsumers? = 60

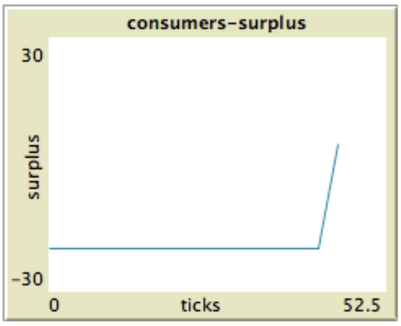
We're going to explore two different levels of PricePreference: 0, 10.

- If we are in a situation in which "PricePreference = 0", the effect on the consumer's mood is straightforward: they will be unhappy from the starting point until the end. T he consumers' surplus will jump when the cheat and the punishment happen, the firms have deeply reduced the price but not enough. Our consumers in this setting are really hard to please.



- If we are in a situation in which "PricePreference = 10, consumers do not appreciate the initial price and they are clearly unsatisfied; then the discount factor and the cheater firm make them happy. The cheating price represents the last price which makes them unhappy because it is anyway higher, then the social welfare greatly increase when the punishment happens because we're finally come to the competitive price. The consumers' surplus is finally positive.



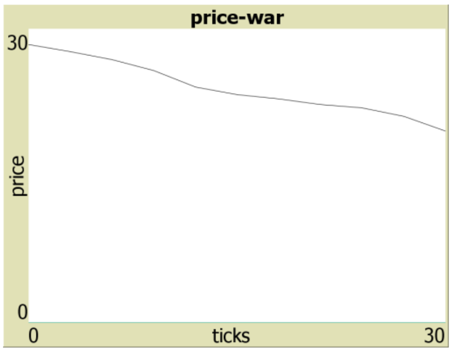


Perfect competition

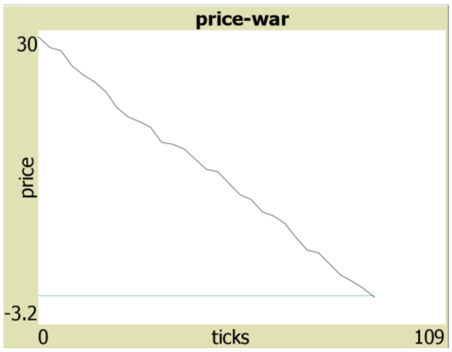
Also in the case of perfect competition we can make some experiments in order to discover the effect of the changes of the price level on the firms' welfare and on the consumers' behavior. Clearly, the possibility of finding different kinds of results in the perfect competition case is reduced if compared with the collusion case, since the delta parameter's issues are offset in this context. Despite of this fact, we can find anyway interesting effects by changing or ruling out the main variables of the model.

Initial price and marginal cost

The price-war that takes place in the perfect competition case has first of all an impact in time on the basis of the initial price, which in our model can be set from 0 to 30, and on the basis of the marginal cost of producing one unit of the product. It is straightforward that the *repeated perfect competition game* will be longer in time as the result of the difference "price – marginal cost" will increase.
In a limit case in which we have "initial price = 30" and "marginal cost = 0", repeated price changes will cover a larger period of time before reaching the final result: "competition-price = 0", and so "price = marginal cost.



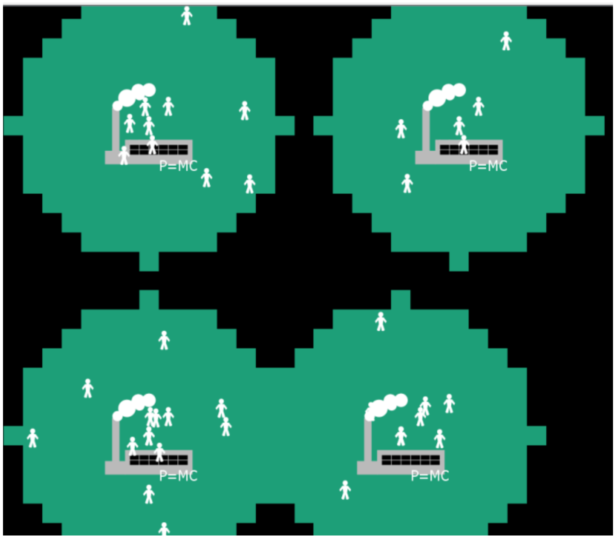
InitialPrice = 30   MarginalCost = 20



InitialPrice = 30   MarginalCost = 0

We can concentrate ourselves also in two other cases, with which we have dealt also in the collusion case: the initial situations of "P = MC" and "P < MC"

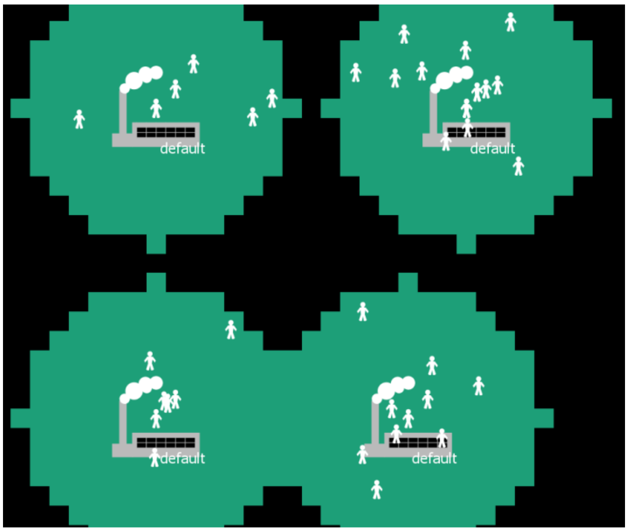
- If we set "initial price = marginal cost", the perfect competition game and the price war among the firms are offset. The companies are in a situation in which they can't lower the price to gain market power, because they would incur in losses, so the game doesn't start.
  This case is particularly clear in our model: when the sliders of "InitialPrice" and "MarginalCost" are set at the same value, the firms show the label "P = MC" and the price competition is ruled out.



InitialPrice = 15   MarginalCost = 15

- If we set "initial price < marginal cost", the perfect competition game is again offset, since the firms are incurring in losses and they are not able to produce. In our model companies cannot have external financing sources, for example by the financial markets or the banking system, so the firms are virtually forced to exit the market, since there is no way to come out from this situation.

Graphically, when "InitialPrice" is minor than "MarginalCost", the firms show the label "default".



InitialPrice = 19   MarginalCost = 20

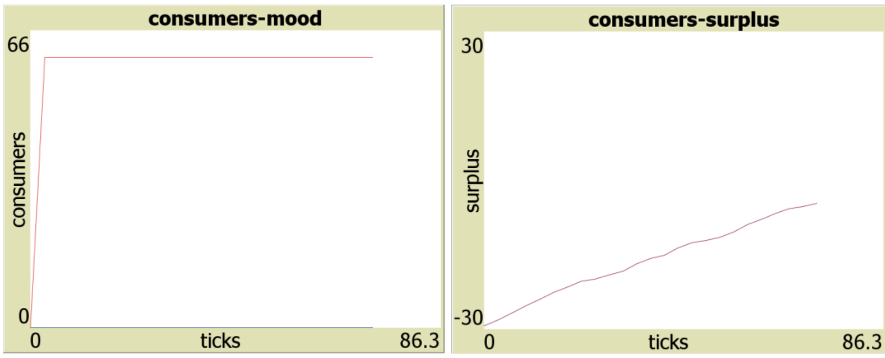
Price preference cases

By imposing different levels of consumers' price preference, we are able to find the effect of the price game on the consumers' surplus and on the consumers' mood.
The experiment is based on the following settings:

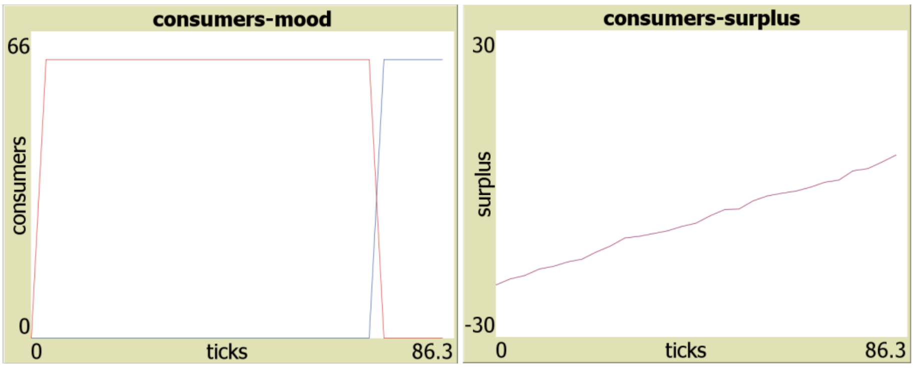
- InitialPrice = 30
- MarginalCost = 5
- HowManyConsumers? = 60

We're going to explore three different levels of PricePreference: 0, 10, 20.

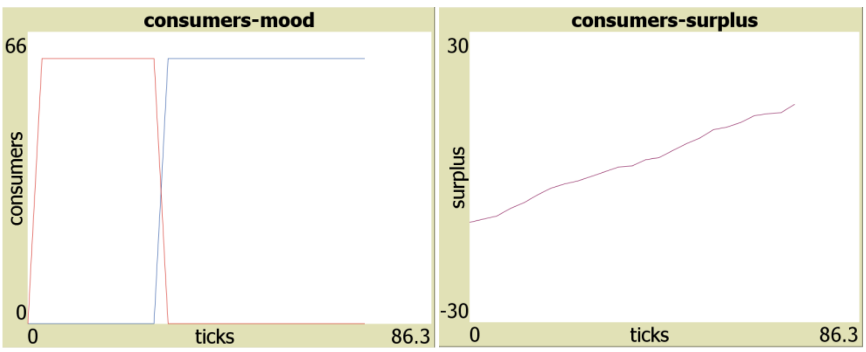
- If we are in a situation in which "PricePreference = 0", the effect on the consumer's mood is straightforward: they will be unhappy from the starting point until the end of the price competition. Moreover, as we can see by the plot, the consumers' surplus (PricePreference – InitialPrice) will increase as the price decreases, but it will stay below the zero level (= - 5).



- If we are in a situation in which "PricePreference = 10", the effect on the consumers' mood is positive from the point in which we have "price preferefence = competition price", so when the price level is equal to 10. Now, the consumers' surplus reaches a positive (= 5) value and consumers are happy (they're unhappy from "price = 30" to "price = 11").



- If we are in a situation in which "PricePreference = 20", the effect on the consumers' mood is and on the consumers' surplus is positive from the point in which the price level is equal to 10. They're unhappy from "price = 30" to "price = 21" and the surplus trend is equal to 15 at the end of the price war.



Conclusions and possible extensions of the model

The main purpose of the model was to reproduce a dynamic-price system in order to demonstrate that, by relaxing the “one-shot game” assumption, the Bertand Paradox can be escaped. In order to do this we have considered the case of perfect competition and the case of collusion, and we have analyzed for the first issue the price-war among the firms of the model, while for the second one the possibility of cheating induced by the “delta” discount factor.
For both the two cases, we have made some experiments by modifying the main variables of the model, initial price and marginal cost, and we’ve showed their different impact on the firms.
Moreover, we’ve introduced a simplified utility function of the consumers in order to find the impact of the market price on their satisfaction, both in the case of collusion and perfect competition. By making experiments, we’ve found that a simple assumption of “happiness/unhappiness”, imposed by a price preference, can lead to very interesting results in terms of “consumers’ surplus”, also in a market with homogeneous goods.

The model, as we’ve pointed out before, represents just a simplification of the market which we can see in the real world. Our purpose was to keep the main assumptions of the Bertand Paradox, so the model is open to a lot of extensions from the firms’ point of view, for example by relaxing the “homogeneous goods” and the “capacity constraints” assumptions.
Another possible extension of the model can come out by setting a more complex utility function of the consumers. We know that in the real world consumers’ preferences are not constant, they change every day, so there are many ways to find interesting results by starting from the model.

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